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# Phase signatures in laser-assisted electron-atom collisions

A. Makhoute[1,2,a] and D. Khalil[1]

[1] UFR de Physique du Rayonnement et des Interactions Laser-Matière, Faculté des Sciences, Université Moulay Ismail, B.P. 11201, Zitoune, Meknès, Morocco
[2] The Abdus Salam International Centre for Theoretical Physics, strada costiera II, 34100 Trieste, Italy

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**Abstract.** We study electron-atom scattering in the presence of a laser field with elliptic polarization. We discuss the dependence of the differential cross sections for the cases of circular and linear polarizations as a function of scattering angle. Interesting typical signatures of the phase between the two components of the circular polarization of the laser field appear in the differential cross section.

**PACS.** 03.75.Pp Atom lasers – 34.50.Rk Laser-modified scattering and reactions – 34.80.Qb Laser-modified scattering – 32.80.Cy Atomic scattering, cross sections, and form factors; Compton scattering

## 1 Introduction

The detailed investigation of atomic processes through interaction radiation has been greatly facilitated by the increased availability of lasers. In addition, a variety of new phenomena have been observed, remarkable by their non-linearity. While multiphoton processes, in particular, multiphoton ionization and harmonic generation have received most attention, laser-assisted electron-atom collisions has not been widely studied. These processes are however of fundamental interest and are important, for instance in the laser heating of plasmas and high-power gas lasers. One of its most remarkable features is the possibility of exciting the target via the absorption of one or more photons.

Experimentally, laser-assisted electron scattering processes have recently become feasible. Several experiments have been performed, in which the exchange of one or more photons between the electron-target and laser field has been observed in laser-assisted elastic [1,2] and inelastic scattering [3–8]. In particular, the excitation process has been widely investigated in the literature by several authors [9,10], mainly in the perturbative (weak-field) limit. The first theoretical studies on the inelastic scattering were inspired by the pioneering works [11–13], in which the laser parameters such as intensity, frequency and also polarization gives interesting results and considerably enriches the study of the collision process. The influence of the laser polarization in the laser-assisted collisions has attracted a great deal of attention in both theoretical and experimental works. It is the purpose of the present paper to investigate, in particular, the role of this parameter in the case of inelastic scattering in helium.

[a] e-mail: makhoute@netcourrier.com

In this paper, we shall present the role of the phase between two components of the circular polarization of the laser field in electron helium collisions using a detailed calculation of differential cross sections. The interaction between the field and projectile is treated in a non-perturbative way by using Volkov waves [14]. On the other hand, the laser-atom interaction, leading to the dressing of the atomic bound states, is treated pertubatively, up to first order. We have performed an "exact" evaluation of the needed infinite sum-over-states, based on simplified hydrogenic functions of the excited spectrum of helium. In order to confirm our numerical results, we have performed the calculations as in our previous paper [15], by two different methods both based on the Sturmian basis expansion.

The paper is structured as follows. In Section 2 we present the general formation of laser-assisted inelastic electron-atom collisions in the elliptical polarization. An account is then given of the techniques that we have used to evaluate the scattering amplitudes. Section 2 also contains a detailed of our numerical results as well as their physical interpretation and interest, and Section 3 concludes the paper. Atomic units (au) are used throughout this paper.

## 2 Theory, results and discussion

Following our previous works [16,17], we shall assume that the laser field is treated classically as single mode and spatially homogeneous, which means that it varies little over the atomic range and that the dipole approximation is valid. Working in the Coulomb gauge, we have for the vector potential of a field propagating along the $\hat{\mathbf{z}}$-axis and represented in the collision plane $(\hat{\mathbf{x}} - \hat{\mathbf{y}})$

$$\mathbf{A}(t) = A_0 \left[\hat{\mathbf{x}} \cos(\omega t + \varphi) + \hat{\mathbf{y}} \sin(\omega t) \tan\left(\frac{\eta}{2}\right)\right], \quad (1)$$

with the corresponding electric field

$$\mathcal{E}(t) = \mathcal{E}_0 \left[\hat{\mathbf{x}} \sin(\omega t + \varphi) - \hat{\mathbf{y}} \cos(\omega t) \tan\left(\frac{\eta}{2}\right)\right], \quad (2)$$

where $\mathcal{E}_0 = \omega A_0/c$, $\mathcal{E}_0$ and $\omega$ are the peak electric field strength and the laser angular frequency, respectively. Here $\eta$ measures the degree of ellipticity of the field and we have the particular cases of linear polarization ($\eta = 0$) and circular polarization ($\eta = \frac{\pi}{2}$) are easily recovered. Here $\varphi$ denotes the initial phase of the laser field. We can recast the electric laser field in terms of its spherical components by

$$\mathcal{E}(t) = \mathcal{E}_0 \sum_{\nu=\pm 1} i\nu \hat{\varepsilon}_\nu \exp(-i\nu(\omega t + \varphi)), \quad (3)$$

where $\hat{\varepsilon}_\nu = \frac{1}{2}[\hat{\mathbf{x}} + i\nu\hat{\mathbf{y}} \tan(\frac{\eta}{2})]$ is the polarization vector.

The analysis of the inelastic electron-atom scattering processes in the presence of the laser field, starting with direct scattering. The Hamiltonian of the electron-atom system in the presence of the laser field may be written in the direct arrangement channel as

$$H = H_F + H_T + V_d, \quad (4)$$

where $H_F$ and $H_T$ are respectively, the Hamiltonian of the free electron and the target in the presence of the laser field, and $V_d$ is the electron-atom interaction potential in the direct channel. We have in atomic units

$$V_d(\mathbf{r}_0, \mathbf{X}) = -\frac{Z}{r_0} + \sum_{j=1}^{Z} \frac{1}{|\mathbf{r}_j - \mathbf{r}_0|}, \quad (5)$$

where $Z$ is the atomic number of the atom, $\mathbf{r}_0$ is the coordinate of the projectile electron, $\mathbf{X}$ denotes the ensemble of target coordinates $\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_Z$. To first-order in the electron-atom interaction potential in the direct channel $V_d$, the $S$-matrix element for direct inelastic scattering from the ground state of energy $E_0$ to a final state of energy $E_f$, in the presence of laser field is given, in the First-Born Approximation (FBA), by the expression

$$\mathbf{S}_{f,0}^{B_1} = -i \int_{-\infty}^{+\infty} dt \langle \chi_{k_f}(\mathbf{r}_0, t) \times \varPhi_f(\mathbf{X}, t) | V_d(\mathbf{r}_0, \mathbf{X}) | \chi_{k_0}(\mathbf{r}_0, t) \varPhi_0(\mathbf{X}, t) \rangle. \quad (6)$$

Here $\chi_{k_0}(\mathbf{r}_0, t)$ and $\chi_{k_f}(\mathbf{r}_0, t)$ are respectively the Volkov wevefunctions of the incident and scattered electrons in the presence of the laser field. $\varPhi_0(\mathbf{X}, t)$ and $\varPhi_f(\mathbf{X}, t)$ are respectively the dressed atomic wevefunctions describing the initial (fundamental) and final states. $\mathbf{k_0}$ and $\mathbf{k_f}$ are respectively the wavevectors of the incident and scattered electrons in the presence of laser field. After integration on the time variable, we can recast equation (6) in the form

$$S_{f,0}^{B_1} = i(2\pi)^{-1} \sum_{\ell=-\infty}^{+\infty} \delta(E_{k_f} + E_f - E_{k_0} - E_0 - \ell\omega) e^{i\ell\gamma_K} f_{f,0}^{B_1,\ell}(\mathbf{K}), \quad (7)$$

with

$$\gamma_K = \arctan\left(\frac{\mathbf{K} \cdot \hat{\mathbf{y}}}{\mathbf{K} \cdot \hat{\mathbf{x}}} \tan(\eta/2)\right), \quad (8)$$

which is particularly important for taking into account the effects of the laser polarization on the variations of the laser-assisted differential cross sections, where $\ell$ is the number of photons emitted during the collision, so that positive values $\ell$ corresponding to absorption and negative ones to emission and momentum transfer $\mathbf{K} = \mathbf{k}_0 - \mathbf{k}_f$ which relatively small. The first Born scattering amplitude, $f_{f,0}^{B_1,\ell}(\mathbf{K})$, is corresponding to the process $0 \longrightarrow f$ accompanied by the transfer of $\ell$ photons can be split in an electric and an atomic amplitudes, corresponding to situations in which the laser field interacts only with the projectile or with the projectile and the target, respectively. These electronic and atomic contributions can be written as [11,16]

$$f_{f,0}^{B_1,\ell}(\mathbf{K}) = f_{elec}^{B_1,\ell}(\mathbf{K}) + f_{atom}^{B_1,\ell}(\mathbf{K}), \quad (9)$$

with

$$f_{elec}^{B_1,\ell}(\mathbf{K}) = J_\ell(R_{\mathbf{K}}) f_{f,0}^{B_1}(\mathbf{K}) \quad (10)$$

and

$$f_{atom}^{B_1,\ell}(\mathbf{K}) = f_1(\mathbf{K}) + f_2(\mathbf{K}), \quad (11)$$

where

$$f_1(\mathbf{K}) = -\frac{i}{K^2} \sum_n \left( \frac{J_{\ell+1}(R_K) e^{i\gamma_K} M_{n0}^-}{\omega_{n0} + \omega} - \frac{J_{\ell-1}(R_K) e^{-i\gamma_K} M_{n0}^+}{\omega_{n0} - \omega} \right) f_{f,n}^{B_1}(\mathbf{K}) \quad (12)$$

and

$$f_2(\mathbf{K}) = -\frac{i}{K^2} \sum_n \left( \frac{J_{\ell-1}(R_K) e^{-i\gamma_K} M_{fn}^+}{\omega_{fn} + \omega} - \frac{J_{\ell+1}(R_K) e^{i\gamma_K} M_{fn}^-}{\omega_{fn} - \omega} \right) f_{n,0}^{B_1}(\mathbf{K}). \quad (13)$$

In both last equations we have

$$R_K = \alpha_o \Big[ (\mathbf{K} \cdot \hat{\mathbf{x}})^2 + (\mathbf{K} \cdot \hat{\mathbf{y}})^2 \tan^2(\eta/2) \Big]^{1/2}, \quad (14)$$

and

$$M_{n'n}^{\pm} = \mathcal{E}_0 \langle \psi_{n'} | \hat{\varepsilon}_\pm \cdot \mathbf{r} | \psi_n \rangle. \quad (15)$$

is the dipole matrix element. $\alpha_o = \mathcal{E}_0/\omega^2$, $\omega_{n'n} = E_n - E_{n'}$ is the Bohr frequency and $J_\ell$ is an ordinary Bessel function of order $\ell$. $\psi_n$ is a target state of energy $E_n$ in the absence of an external field. $f^{B_1}_{f,0}(\mathbf{K})$, $f^{B_1}_{n,0}(\mathbf{K})$ and $f^{B_1}_{f,n}(\mathbf{K})$ are the first-Born amplitudes corresponding to the scattering process $0 \to f$, $0 \to n$ and $n \to f$ without laser field.

The first-Born differential cross sections for electron-atomic collisions with the transfer of $\ell$ photons is given by

$$\frac{d\sigma^{\ell}_{f,0}}{d\Omega} = \frac{k_f}{k_0} |f^{B_1,\ell}_{f,0}(\mathbf{K})|^2. \tag{16}$$

The main problem in evaluating the scattering amplitudes corresponding to the first-order contributions to the $S$-matrix element for laser-assisted elastic scattering and excitation process, consists of performing the summation over the intermediate states. In order to calculate exactly the corresponding radial amplitudes without further approximation, we have used two different methods based in Sturmian approach similar to the ones described in our previous works [15,16]. This approach allows us to take into account exactly the bound-continuum-state contributions, which are of crucial importance for electron impact excitation at intermediate energies. These methods of computation constitute an important advantage in the present context as compared to earlier ones relying on the closure approximation [11].

The contribution for laser-assisted inelastic collisions to the $S$-matrix of exchange scattering which leads to some conceptual difficulties but would not significantly alter the results of the present discussion. We have consider in the present paper only the leading term of $g^{B_1,\ell}_{f,0}$, the exchange amplitude for electron-atom collisions with the transfer of $\ell$ photons used in reference [11]. It is known that the exchange effects in collisions are important at low relative velocities, while the FBA is an essentially high-energy approximation. Thus, the first-Born differential cross section corresponding to the inelastic scattering process, with the transfer of $\ell$ photons, is given by

$$\left(\frac{d\sigma^{\ell}_{el}}{d\Omega}\right) = \frac{k_f}{k_0} \mid f^{B_1,\ell}_{f,0} - g^{B_1,\ell}_{f,0} \mid^2, \tag{17}$$

does not depend on the initial phase $\varphi$ of the laser field, due to the inability of the collision time to be defined, as a result of the approximation of the projectile wave packet by a mono energetic beam of infinite duration [18].

The formalism described above has been applied to various kinds of scattering process involving a neutral atom, in particular for helium in the initial state: elastic collision and excitation. In every case, the results that have been obtained show the importance of the role played by the dressing of the atomic states by the laser field, specially at small-scattering angles, and the importance of light polarization effects, namely, a circularly polarized laser can give larger cross sections than a linearly polarized one, by several orders of magnitude [10,16,17].

The present semiperturbative method with the Sturmain basis expansion takes into account the target atom distortion induced by the presence of a laser field. The validity of our treatment is based on the fact that the laser-atom target interaction is non resonant. This condition become stringent if the laser frequency is comparable to any characteristic atom transition frequency. We note that the inelastic scattering process can be considered as non resonant if for a given frequency, the intensity does not exceed a certain limit [16]. Such a condition will be respected by our choice of the Nd-YAG laser frequency $\omega = 1.17$ eV and $\mathcal{E}_0 = 10^7$ Vcm$^{-1}$. In the calculation of the amplitudes (10), (12) and (13), we need to know the explicit form of the atomic wave functions in the absence of an external field. For the ground state of helium and for $n^1$S, $n^1$P, $n^1$D we use the wave functions proposed in reference [13]. We note that the doubly excited states are neglected in view of the weak contribution of these states to the elastic process [11,16].

We will illustrate and discuss the effects of the phase between the two components of the circular polarization of the laser field in the elastic collision and excitation process of helium target by a fast electrons in the presence of a laser field. In helium, there have been comparatively fewer attempts to address the role of the dressing of the atomic states by the strong laser field, which not completely answered yet, the main reason being that the computation is much more complex. This is unfortunate in view of the fact that helium would lend itself more easily than hydrogen, to experimental verifications. Note however that, though simplified, the model contains all the ingredients needed for the discussion of the physics of such processes.

The results presented in this paper are obtained for a geometry in which the wavevector $\mathbf{k}$ being contained in the scattering plane coplanar circular polarization(CCP): see figures in Bouzidi et al. (1999), where free-free differential cross sections are maximum at a particular laser intensity and incident electron energy [16,17], and where the laser-assisted differential cross section depends on the orientation of the polarization unit vector $\hat{\varepsilon}_\nu$ and on the phase between the two components of the laser field. We compare our results in coplanar circular polarization with the results obtained for a geometry in which the polarization vector of the field $\mathcal{E}_0$ is parallel to the direction of incoming electron wave vector $\mathbf{k}_0$ ($\mathcal{E}_0 \parallel \mathbf{k}_0$), where the differential cross section in presence of the laser field only depends on the orientation of the polarization unit vector $\hat{\varepsilon}_\nu$ and with the values obtained by using the Kroll-Watson approximation (KWA), where the differential cross sections for exchange of photons are related to the field-free differential cross section ($\frac{d\sigma}{d\Omega}$) through

$$\frac{d\sigma^{\ell}}{d\Omega} = \frac{k_f}{k_0} J^2_{\ell}(\lambda) \frac{d\sigma}{d\Omega}. \tag{18}$$

The motivation for this particular choice is that the electronic term $f^{B_1,\ell}_{elec}(\mathbf{K})$ is identical for both geometries, because the argument of the Bessel functions reduces to the same value $R_K = \alpha_o(k_i - k_f \cos\theta)$ in these two cases, where $\theta$ is the scattering angle. The same situation occurs when we compare the differential cross sections corresponding to the laser wavevector $\mathbf{k}$ being perpendicular to the scattering plane (perpendicular circular polarization, PCP) with those obtained for linear polarization

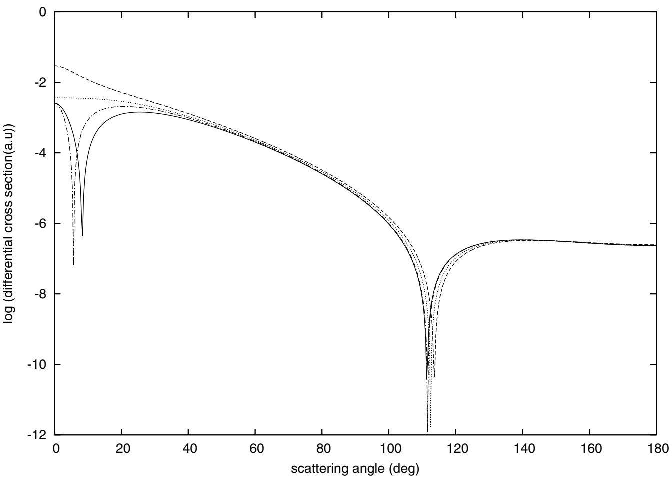


**Fig. 1.** First Born differential cross section corresponding to the excitation of $2^1$S state of helium, with the absorption of one photon ($\ell = 1$), as a function of the scattering angle $\theta$. The incident electron energy is 100 eV, the laser frequency is 1.17 eV, and the electric field strength is $10^7$ V/cm. Dotted line: the KWA results. Dotted-dashed line: linear polarization ($\mathcal{E}_0 \parallel \mathbf{k}_0$). Dashed line: circular polarization (CCP with $\gamma_K = 0$). Solid line: circular polarization (CCP with $\gamma_K = \pi$).

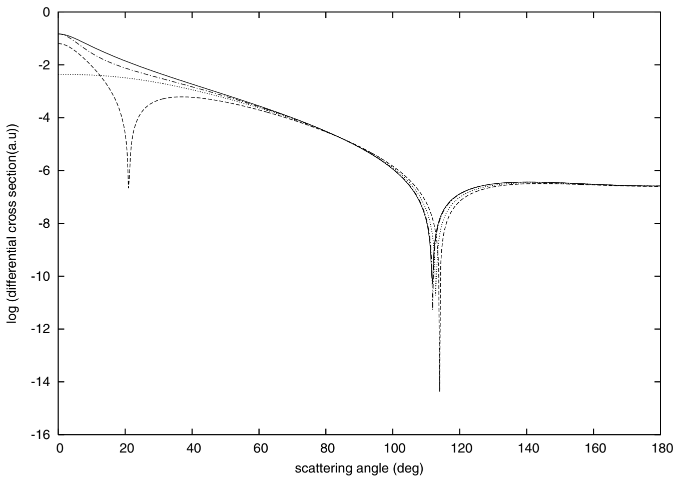


**Fig. 2.** As in Figure 1, but with the emission of one photon ($\ell = -1$).

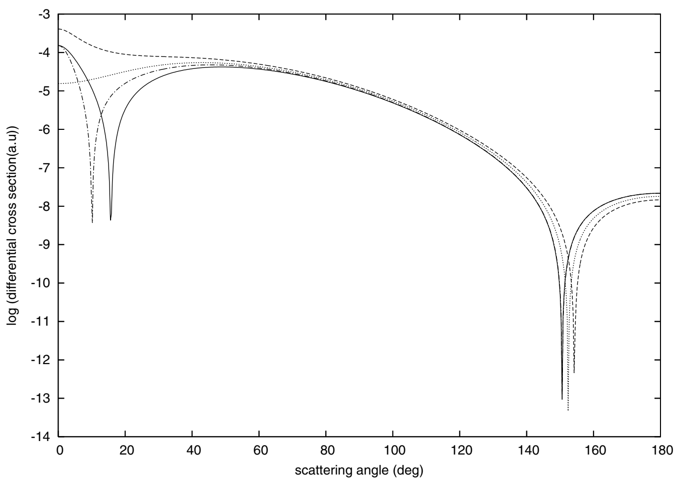


**Fig. 3.** As in Figure 1, but with the absorption of two photons ($\ell = 2$).

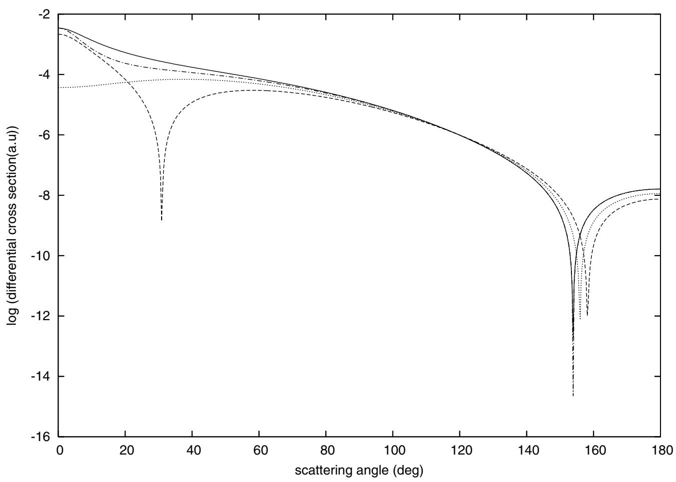


**Fig. 4.** As in Figure 2, but with the emission of two photons ($\ell = -2$).

corresponding to geometry where polarization vector of the field is taken to be parallel to the momentum transfer $\mathbf{K}$ ($\mathcal{E}_0 \parallel \mathbf{K}$), the argument of the Bessel function being then reduced to an identical value $R_K = \alpha_o K$, but there is no effect of phase between the two components of the circular polarization perpendicular of the laser field [17]. In both cases, the differences observed in the angular dependence of the differential cross sections are signatures of effects arising from the dressing of the target.

In the case of elastic collision, as noted before, a circularly polarized laser can give larger cross sections then a linearly polarized one by several orders of magnitude. The absence of deep minima in the differential cross sections for circular polarization due to the presence of a complex phase forbids the occurrence of a complete destructive interference between the two amplitudes [16]. For linear polarizations ($\mathcal{E}_0 \parallel \mathbf{k}_0$ and $\mathcal{E}_0 \parallel \mathbf{K}$), some of these minima are of purely kinematics origin, another kind of minima associated with destructive interference between electronic and atomic amplitudes. These minima do not appear in the case of circular polarizations (PCP and CCP), as the atomic contribution, equation (11), becomes complex, which impedes the occurrence of the cancellation with the electronic term, equation (10). This partially accounts for the fact that the magnitude of the cross section can be larger for circularly polarized light than for linear polarization. These results demonstrate the importance of the effects of the light polarization in this class of free-free transitions at small scattering angles and are present in hydrogen as well as in helium, while the effects of the phase between the components of the circular polarization of the laser field is absent.

In Figures 1–4, we present the differential cross sections for laser-assisted excitation of the $2^1$S with the net exchange of up two photons ($\ell = \pm 1, \pm 2$) as a function of the scattering angles $\theta$. In each of these figures, we have displayed the excitation amplitude for two different geometries: for linear polarization $\mathcal{E}_0 \| \mathbf{k}_0$ and for circular polarization CCP. The reason for this particular choose is that $\tan(\gamma_K)$ vanishes, so that $\gamma_K = 0[\bmod \pi]$. These values of $\gamma_K$, in a interval $[0, 2\pi[$, correspond to the case where the two components of the electric field in the plane $\widehat{\mathbf{y}}, \widehat{\mathbf{x}}$ vary as a function of time, with the same phase $\gamma_K = 0$ or with opposite phases $\gamma_K = \pi$. This behavior can be explained by the change of Bessel functions from CCP with $\gamma_K = \pi$ to CCP with $\gamma_K = 0$, making a change of sign of the atomic, i.e. the phase-dependent factor $\exp(\pm i\gamma_K)$ present in the atomic term changes the sign of its real part. The change of phase is absent in the case of circular polarization PCP, because $\tan(\gamma_K) = 0$ for $\gamma_K \neq 0$ and its unique.

In Figures 1–4 we show the laser-assisted differential cross sections corresponding to the $1^1\text{S} \longrightarrow 2^1\text{S}$ excitation process. The complete results obtained by using the scattering amplitude equation (18) for two polarizations is compared with the "KWA" cross section in which dressing effects are neglected. According to the domain of validity of the treatment used for taking into account the laser-atom interaction, the Nd-NAG laser frequency will be taken to be $\hbar\omega = 1.17$ eV (0.043 au) and $\mathcal{E}_0 = 10^7$ Vcm$^{-1}$. These parameters are such that the target dressing gives the small momentum transfer $\mathbf{K}$, the differential cross section corresponding to the electronic term much smaller than that in the absence of laser field for $|\ell| \geq 1$ and the soft photon approximation is not valid [19]. As noted before the inclusion of higher order terms of the direct scattering matrix and of exchange only increases the results at small scattering angles but does not change the qualitative difference between the "electronic" and complete (electronic and atomic) results with net transfer of photons we can see that for angles below 40° for absorption and below 60° for emission there are important differences between the two polarizations. Indeed, as in the case of elastic collision, dressing effects are shown to be dominant in the forward direction for linear polarization where $\mathcal{E}_0 \parallel \mathbf{k}_0$ and circular polarization CCP and for larger scattering angle for circular polarization coplanar. In both cases, the differences observed in the angular dependence of the cross sections results from the differences between the contributions of the atomic terms, i.e on the dressing of the target. One observes indeed strong modifications of the cross sections, as compared with the results of calculations in which dressing effects are neglected (KWA). For the laser frequency and field strength chosen here these modifications occur at $\theta \leq 40°$. In addition, notable differences are observed, depending on the polarization state of the light and on the phase between two components of the circular polarization of the laser field. The overall magnitude of cross section corresponding to difference between circular polarization and linear polarization is more important and clearly observed when we compare the differential cross for linear polarization $\mathcal{E}_0 \parallel \mathbf{k}_0$ with circular polarization coplanar with $\gamma_K = 0$ for absorption and with $\gamma_K = \pi$ for emission. This difference decreases when $|\,\ell\,|$ increases. This behavior is particularly important from the experimental point of view since it is, in principle, easier to measure the laser-assisted differential cross sections amplitudes for larger scattering angle and for such a choice of phase, where the dressing effects of the target contribute significantly.

As indicated in our previous paper on elastic scattering of helium [16] and excitation of atomic-hydrogen [17], we have observed the existence of two kinds of minima ($m_1$) and ($m_2$) corresponding respectively, to the situations when $f^{B_1,\ell}_{elec}(\mathbf{K}) + f^{B_1,\ell}_{atom}(\mathbf{K}) = 0$, which a destructive interferences (the electronic and the atomic amplitudes are varying in opposite directions when the momentum transfer increases), and at angles such that the argument $R_K$ of the Bessel functions actually vanishes. This last minimum exists for absorption, with net exchange of photons, in the cases when $\mathcal{E}_0 \parallel \mathbf{k}_0$ and CCP with $\gamma_K = \pi$ and for emission in the case CCP with $\gamma_K = 0$. We notice that the shape of the differential cross section in the case of coplanar circular polarization with $\gamma_K = \pi$ follows the same behavior that corresponds to the results obtained in the case of linear polarization. The presence of a destructive interference between the electronic and the atomic amplitudes is a general feature of $1^1\text{S} \longrightarrow n^1\text{S}$ transitions in the case of absorption $\ell > 1$ for $\mathcal{E}_0 \parallel \mathbf{k}_0$ and CCP with $\gamma_K = \pi$ and emission $\ell < 1$ in the case CCP with $\gamma_K = 0$. This is due to the presence, in the atomic terms of $s-p$ transition amplitudes, which behave like $K^{-1}$ for small $K$. This behavior can be explained by the change of Bessel functions from absorption $\ell = 1, 2$ to emission $\ell = -1, -2$, making a change of sign of the atomic amplitudes.

## 3 Conclusion

Our results show that, everything else being fixed, a circularly polarized laser (CCP with $\gamma_K = 0$ for absorption and CCP with $\gamma_K = \pi$ for emission) can give larger cross sections than a linearly polarized one, by several orders of magnitude. This is one interesting typical signatures of the phase between the two components of the circular polarization of the laser field in the differential cross section. This behavior should constitute an interesting and attractive point for the experimentalists to measure the cross amplitudes in the case of a circularly polarized laser beam and for such choice of phase.

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